- 1. Error Bars. Two new concrete mixes are tested, resulting in the following data. Mix 1: sample size = 15, sample mean = 4900 psi, sample standard deviation 400 psi. Mix 2: sample size = 31, sample mean = 5000 psi, sample standard deviation 150 psi.
  - a. What significance level is associated with a 90 % Confidence level?
  - b. Determine the 90 % Confidence Interval for each mix. State them using the proper format. Show your work. If it is appropriate to use the Z distribution, do so.
  - c. Plot the means and error bars (90 % CI) in a column chart using Excel. Use proper chart formatting.
  - d. Do the error bars give you any statistical information regarding the difference between the mix means? If so, what?

## **SOLUTION**

a.  $\alpha$  = 0.1

b. Use  $\alpha/2 = 0.05$  because Cl's are 2 sided. Use T for mix 1, z for mix 2 Mix 1:

• 
$$U_1 = \mu + t_{0.05,14} \cdot \frac{s}{\sqrt{n}} = 4900 + 1.76131 \cdot \frac{400}{\sqrt{15}} = 5082 \text{ psi}$$

• 
$$L_1 = \mu - t_{0.05,14} \cdot \frac{s}{\sqrt{n}} = 4900 - 1.76131 \cdot \frac{400}{\sqrt{15}} = 4718 \text{ psi}$$

• (4718 psi < μ < 5082 psi) with 90 % Confidence

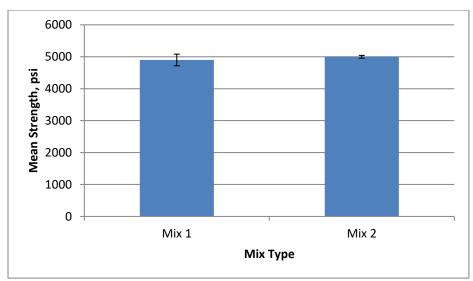
Mix 2:

$$\begin{array}{ll} \bullet & U_2 = \ \mu + t_{0.05,30} \cdot \frac{s}{\sqrt{n}} = \ 5000 + 1.644854 \cdot \frac{150}{\sqrt{31}} = 4956 \ \mathrm{psi} \\ \bullet & L = \ \mu - t_{0.05,30} \cdot \frac{s}{\sqrt{n}} = \ 5000 - 1.644854 \cdot \frac{150}{\sqrt{31}} = 5044 \ \mathrm{psi} \end{array}$$

• 
$$L = \mu - t_{0.05,30} \cdot \frac{s}{\sqrt{n}} = 5000 - 1.644854 \cdot \frac{150}{\sqrt{31}} = 5044 \text{ ps}$$

• (4956 psi < μ < 5044 psi) with 90 % Confidence

c.



d. The confidence interval error bars overlap; thus, one cannot say anything about the difference between the two mix means.

- 2. Sample Size: Determine the sample size needed to estimate the density of a new asphalt mix with a precision of 3 kg m<sup>-3</sup> if the population standard deviation is UNKNOWN and the sample standard deviation is assumed to be 6 kg m<sup>-3</sup>. Use a significance level of 0.05.
  - a. What is the confidence level?
  - b. What sample size is estimated using the Z distribution? Can the Z distribution be used? Should the T distribution be used? Explain.
  - c. If appropriate, what sample size is estimated using the T distribution? Document your work in a table, with sample calculations shown below the table.

## SOLUTION

a. Confidence =  $1 - \alpha = 1 - 0.05 = 0.95$  or 95 %

b. Using the Z distribution, 
$$N=\left(rac{z_{lpha/2}\sigma}{P}
ight)^2=\left(rac{1.960\cdot 6}{3}
ight)^2=15.4$$

Because the population standard deviation is unknown and the required sample size < 30, the Z distribution should not be used.

c. Using the T distribution,  $N=\left(\frac{t_{\alpha/2,\nu}s}{P}\right)^2$ , solve by trial and error.

N Guess	t <sub>0.025,n-1</sub>	N Calculated
16	2.131	18.2
17	2.120	18.0
18	2.110	17.8
17.98	1.120	17.98

Second row sample calculation: N guess = 17, 
$$t_{0.05/2,17-1} = t_{0.025,16} = 2.120 \& \left(\frac{2.120 \cdot 6}{3}\right)^2 = 18$$

Answer: N = 18.

A Solver solution gives N = 17.96 = 18. (Optional)

The T distribution is the correct distribution, as the pop. standard deviation is unknown and the required sample size is < 30.